Algebra II

8-7

Rational Root Theorem

Upper Bound - When the quotient is a row of all positive or all negative numbers. This means there will never be a solution greater than the divisor.

divisor

	2	3	-7	3	-9
3		6	27	60	189
	2	9	20	63	180

← all positive

no solution bigger than 3.

(only works for positive divisors)

Lower Bound - When the quotient is a row of alternating positive and negative numbers. This means there will never be a solution less than the divisor.

	2	3	-7	3	-9
-4		-8	20	-52	196
	2	-5	13	-49	185

← alternating signs

answer cannot be less than -4

(only works for negative divisors)

Rational Root Theorem -

Fraction

$P(x) = 6x^3 - 5x^2 + 7x - 12$

Q $\pm \frac{1}{1}, \frac{12}{1}, \frac{2}{1}, \frac{6}{1}, \frac{3}{1}, \frac{4}{1}$

$\pm \frac{1}{6}, \cancel{\frac{12}{6}}, \cancel{\frac{2}{6}}, \cancel{\frac{6}{6}}, \cancel{\frac{3}{6}}, \cancel{\frac{4}{6}}$

$\pm \frac{1}{2}, \cancel{\frac{12}{2}}, \cancel{\frac{2}{2}}, \cancel{\frac{6}{2}}, \frac{3}{2}, \cancel{\frac{4}{2}}$

$\pm \frac{1}{3}, \cancel{\frac{12}{3}}, \frac{2}{3}, \cancel{\frac{6}{3}}, \cancel{\frac{3}{3}}, \frac{4}{3}$

Steps for solving Polynomials of Degree 3 or greater.

1) Identify the total number of solutions. (Biggest Power)

2) Descartes' rule of signs + | - | i

3) Rational Root Theorem (possible Q 's) (see last slide)

4) Begin looking for either Upper or Lower Bounds

5) Anytime a solution is found, use the quotient as the new dividend.
↳ zero remainder

6) Repeat steps 4 through 6 until both the Upper or Lower Bounds are both found. At any time the polynomial is quadratic, stop and use the methods for solving quadratics (factor, radical, quadratic formula)

7) Once done, try any remaining numbers from the Rational Root Theorem.

8) Round to tenths as a last resort.

Solve.

1) $x^3 - 7x + 6 = 0$ {-3, 2, 1}

Q ± 1, 6, 2, 3

+	-	i
2	1	0
0	1	2

	1	0	-7	6
-1		-1	1	6
	1	-1	-6	12
-2		-2	4	6
	1	-2	-3	12
-3		-3	9	-6
	1	-3	2	0

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

2, 1

Once the equation is quadratic, finish with factoring, radical method, or quadratic formula. Factoring used here.

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3) $x^3 - 3x^2 + 2x - 8 = 0$ ±1,2,4,8

Can't Find

+	-	i
3	0	0
1	0	2

Q ± 1,2,4,8

Negatives were not considered because of DesCartes Rule of Signs.

	1	-3	2	-8
1		1	-2	0
	1	-2	0	-8
2		2	-2	0
	1	-1	0	-8
3		3	0	6
	1	0	2	-2
4		4	4	24
	1	1	6	16

We don't try 8 because Upper Bound is Reached.

Since all options were exhausted, we cannot find the solutions.

11) $3x^4 + 4x^3 - x^2 + 4x - 4 = 0$ $\{-2, \frac{2}{3}, \pm i\}$

+	−	i
3	1	0
1	1	2

Q: $\pm 1, 2, 4$
$\pm \frac{1}{3}, \frac{2}{3}, \frac{4}{3}$

	3	4	−1	4	−4
UB 1		3	7	6	10
	3	7	6	10	6
−1		−3	−1	2	−6
	3	1	−2	6	−10
LB −2		−6	4	−6	4
	3	−2	3	−2	0
$\frac{2}{3}$		2	0	2	
	3	0	3	0	

$3x^2 + 3 = 0$

We found the Upper and Lower Bounds, so now we have to try the rest of the fractions between 1 and -2.

Finish using the radical method to find the last two solutions.

$3x^2 + 3 = 0$
$3x^2 = -3$
$\sqrt{x^2} = \sqrt{-1}$
$|x| = i$
$x = \pm i$

Pg 384

2, 4-8 all,
10, 12, 22-24 all